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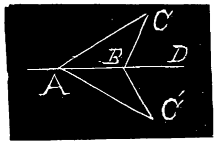
AVERAGE AND PROBABILITY.

70. Proposed by Professor MILLER.

A ship at A observes another at B, whose course is unknown. Supposing their speed the same, prove that the chance of their coming within a given distance, d, of each other is always $(2/\pi)\sin^{-1}(d/a)$, whatever the course taken by A; provided its inclination to AB is not greater than $\cos^{-1}(d/a)$, where $AB=a$. [From *Cambridge Mathematical Tripos*, 1871.]

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $AB=a$, $\angle CAB$ which ship A makes with $AB=\theta$, $\angle CBD$ which ship B makes with $AB=\varphi$ where C is the intersection of the two courses. Then $\angle ACB=(\varphi-\theta)$.



Let v=velocity of each ship, $AC=b$, $BC=c$.

Then in time t, the ship is distant from C, $b-vt$. B is distant from C, $c-vt$.

$\therefore$ $d^2=(b-vt)^2+(c-vt)^2-2(b-vt)(c-vt)\cos(\varphi-\theta)$ ....(1).

Differentiating with reference to t for a minimum, we get

$$v(b-vt)+v(c-vt)=[v(c-vt)+v(b-vt)]\cos(\varphi-\theta).$$

$\therefore$ $t=(b+c)/2v$....(2).

Substituting (2) in (1) we get $d=(b-c)\cos\frac{1}{2}(\varphi-\theta)$.

But $b=a\sin\varphi/\sin(\varphi-\theta)$, $c=a\sin\theta/\sin(\varphi-\theta)$.

$\therefore$ $d=\dfrac{a\cos\frac{1}{2}(\varphi-\theta)(\sin\varphi-\sin\theta)}{\sin(\varphi-\theta)}$.

Now $\sin\varphi-\sin\theta=2\cos\frac{1}{2}(\varphi+\theta)\sin\frac{1}{2}(\varphi-\theta)$.

$\therefore$ $d=a\cos\frac{1}{2}(\varphi+\theta)$.

$\therefore$ $\varphi=2\cos^{-1}(d/a)-\theta=\theta_1$.

Let $\cos^{-1}(d/a)=\beta$.

$$\therefore \text{ Chance }=\frac{\int_{-\beta}^{\beta}\int_{\theta_1}^{2\pi-\theta_1}d\theta d\varphi}{\int_{-\beta}^{\beta}\int_{0}^{2\pi}d\theta d\varphi}=\frac{\int_{-\beta}^{\beta}(2\pi-2\theta_1)d\theta}{4\pi\beta}=\frac{(\pi-2\beta)}{\pi}.$$

$\therefore$ $p=(2/\pi)[\frac{1}{2}\pi-\cos^{-1}(d/a)]=(2/\pi)\sin^{-1}(d/a)$.